webinar 8: Mössbauer spectroscopy

(automated transcription)

So, welcome to this webinar on the monopole shift, which is our first actual manifestation of hyperfine interactions, but before going to the science, a little bit of housekeeping, just a little bit. So some of you mentioned that in the Are You On Board Forum there were several questions asked that actually were questions for the week before, and indeed I realized that there was a link to an old version of the forum, this has now been corrected, so for the people of next year this problem will not appear anymore, and there were one or two places with broken links in the pages, also these have been restored. I noticed that on Zulip there was a little bit of activity, people asking questions there, which is very good, and even better, others of you who answered these questions, so I didn't have to do anything, you were discussing among all of you, so that's nice, so more of that I would say. And that's it for the housekeeping, still no mentioning on the chat, or do I don't see the chat? No, I have it here. Okay, well, would there be problems, please put it in the chat, although, well, good. Our scientific topic for today, the monopole shift, so as I said before, the first experimental observational realization of the presence of hyperfine interactions. We discussed hyperfine interactions already quite a bit by now, but how do we notice that they are there, and the monopole shift is the first place where we will see this. The module started with a short summary of the framework, and you will see this in every module of the first half of the course from now on, there will always be this short summary. Of course, if you have studied the previous module, the one on the framework, you don't need to look at this one again, it's exactly the same content, only summarized, and the reason for that is that this course is also meant to be taken in a modular way. Imagine that somebody out there wants to learn something about the monopole shift, but is very much familiar with all other aspects of hyperfine interactions, well, then this person can just take this module, but because it is quite a bit connected to that story of the framework, I always come back to the very important picture number one, therefore it's useful to have that framework summarized as a quick introduction to the context of the module that will follow. So that repetition is not there necessarily for you, but for people who want to take this course in a very modular way. An important part of the present module was this table, where you see at the left-hand side in a vertical direction, so well, let me point to it, here you see our multipole expansion, so we start with the monopole term, then here there is the first non-zero term in the charge-charge interaction, the quadrupole term, then the hexadecupole term, and so on up to infinity. The left column is in the assumption that the nuclear distances are always smaller than the electron distances, so there are no overlap corrections. If there would be overlap corrections, then you have here on this line the overlap corrections to the monopole term, which themselves are an infinite series, here there are the overlap corrections to the quadrupole term, and so on. What is indicated here in green, that is the interaction you have looked at in your courses on atomic physics, molecular physics, solid-state physics, the monopole interaction between the point nucleus and the electron cloud. Everything that is in this table in black are contributions that will be discussed in this course, some of them extensively, others will only be mentioned, but everything that is in black will be met at some point. Everything that is in red is too small to be of any practical significance, and everything that is not mentioned in this table, so the triple dots that always go to infinity, these would also be in red, these are also too small to be of any practical significance. The present module is about this green box, so the first order overlap correction to the monopole term, and if I ask you in the confidence statement, I can draw a scheme to explain how overlap corrections appear as an infinite correction series to the different multiple orders of energies, well that's this scheme, if you understand what I said in the past two minutes, if you understand this scheme, then you can answer yes, I am fully confident on this question. I see that the distribution stays away from number 6, from the fully confident level, so hopefully this extra explanation or repetition will have cured that. I asked you several tasks about this monopole shift, and the first one was, imagine that you are an observer who sits inside the hydrogen nucleus, and who observes the behavior of the 1s electron in the hydrogen atom over time. How would you describe to a 15 year old child the time averaged position of this electron? The question may seem silly and trivial, but it has a good meaning, so I want you to think about what does it mean that the 1s electron is in an orbital with particular characteristics, and by asking you to formulate that in a language that is not the language of quantum physics, but that is everyday language that a 15 year old child would understand, by forcing you to speak in that language, I hope to reveal where there might be misconceptions in your understanding of this electron orbital. And I was happy to see that one of you even tried this in real life, made an explanation and explained it to, in this case, his 15 year old sister, so that's the good spirit, bringing this course in your daily life, that is always great. Well let's look at a few answers, one answer was, most of the time the electron orbits around the nucleus at a constant radius which is called the Bohr radius, and that's indeed the Bohr picture of the hydrogen atom, which for our context here is too classical, so it's okay to think in this picture for things like transition energies between orbitals, but for hyperfine interactions this Bohr picture is too classical. Another answer was, somebody who says, well it looks like the electron is moving inside a hollow sphere with a certain radius, a radius that is very small, and okay, the explanation that is given here, that little story, that was apparently meant to emphasize the small size of that sphere, which is a good message, but we want to understand something more here, we want to get a grasp on how that electron is distributed inside that sphere. Another answer, one that could be correct, or at least that touches one aspect that is correct, the time-averaged position of the 1s electron is like a cloud, where the electron has a higher probability of being close to the center rather than further away, and so that answer is correct, provided you refer to the correct type of probability, so I show here a picture of the radial probability density for the 1s orbital in the hydrogen atom, and that function describes exactly what has been said in that red text, the closer you are to the origin, so the closer the electron is to the nucleus, the higher the probability to find it there. Does that match your intuitive understanding of the hydrogen 1s electron? Maybe not, if you try to reconcile this with the Bohr picture you might have a different intuition, nevertheless this picture is correct, the probability to find an electron at some given volume at a distance r from the nucleus is given by this graph, so the highest probability per unit of volume to find an electron is in the neighborhood of the nucleus. There is another possible answer, however, somebody says, I would shout to my 15 year old sister the radial distribution plot that describes the amount of time the electron spends in an area at increasing distances from the nucleus, so the average position is some 53 picometers away from the center, and that's what is shown on the picture there, you have there not the radial probability density but the radial distribution function, which is the radial probability density multiplied by 4 pi r squared, and that gives you this Bohr radius of 53 picometers as the most probable position to find the electron, that is where the connection with the Bohr atom happens. Now both of these pictures show one aspect of the electron cloud, and it's good to understand that electron cloud in these two different ways. There is one answer, one that was not given this year but that was given a few years ago and that I keep repeating because it really gives a good insight, I don't ask you to read this, I just show it on the screen such that you can either screenshot it now or go back to the video later and read it there, but I will tell this story based on the picture here on the next slide. So how does the story go? You have a tree with only branches, no leaves, and the nucleus is imagined to be in the center of that tree, there where you have the red dot, and now you have a bird that is hopping around in the different branches of the tree, and whenever it lands at a particular place, a leaf emerges at that place. After a while you have many leaves on the tree and now you ask questions about these leaves, and one question would be, if I take a unit of volume, what would be the probability to find a leaf in that unit of volume? And that probability you plot as a function of the distance of that position from the nucleus, that is what the radial probability density expresses. You could however also ask a different question, you could take a thin spherical shell, so not a surface but a volume, but a volume that is a shell, a spherical shell, and you could wonder what is the probability to find leaves in that spherical shell? Now if that spherical shell is close to the nucleus, that volume will be small. If you take such a shell with the same thickness far away from the nucleus, you will have much more volume. So it will be, as the volume is larger, if you are away from the nucleus, the probability to find leaves inside that shell will be larger, but the probability for leaves to appear at that distance is going down, so the net effect will be that this curve will go to zero at large R, but also at small r's, where the probability to find a leaf is highest, well there the shell volume is smallest, so you will start from a low value. And there will be a transition point somewhere at the Bohr radius, there you have a relatively large volume and still a relatively high probability density, and that combines to the radial distribution function that peaks at this distance. So that picture shows, well at least in my impression, this picture shows very well the difference between these two properties. These two different ways to express how the 1s electron is distributed over that spherical orbital. With this in mind, let's look at a few other answers that were partially true. So somebody suggested let's put a camera at the center of a merry-go-round, where the camera is the nucleus, and somebody is at the outside of the merry-go-round, and who is pushing the merry-go-round, and the camera is taking pictures periodically, and this person will appear always at different positions relative to the camera, and your time averaged position of the person will be a circle that is going, a circle with the edge of the merry-go-round. That's a good image, the only problem is that you have just one distance, so it's a bit too classical, but it could be the picture for one particular ring here, so you are taking one vertical slice of this radial distribution function. Another story one of you made was, let's imagine that you are watching, or that you are listening to a soccer match, but you have your eyes closed, because the different players in the team have different roles, they will be active in different areas of the field, and if you just listen to the players shouting to each other, you can have an impression on where these different players are, just as the different electrons of the different orbitals in a hydrogen atom have different positions for the electrons to be, and for the 1s you would have this spherical distribution, so also this picture is right, but we wanted to go one step further, we wanted to see how these electrons are distributed over that spherical cloud, just mentioning that it is a spherical cloud was not yet the full answer we were after. Another analogy, also a nice one, was comparing the electron to a butterfly, who is visiting different flowers in a rather random way, and you cannot predict where the butterfly will go, but you know that the chance to find it near a particular flower is higher than the chance to find it somewhere in a totally different place. So also here we wanted to have more details, we wanted to see where exactly can we find the butterfly. So the overall conclusion from this task, after having thought about the 1s electron in this non-quantum physics way, what should we take with us from this? That the probability to find an electron at the position of the nucleus is not zero. If we think in terms of Bohr atoms, if we think in terms of radial distribution functions that start at zero and that have a peak somewhere far away from the nucleus, we tend to forget that this is possible. But it is possible that electrons, and then in particular s-electrons, happen to be at the position of the nucleus, even inside the nucleus, because the position of the nucleus, that's one point, but at that one point, at the origin of the axis system, there the s-electrons have a non-zero probability to appear. The nucleus is a volume, is a sphere, so this origin is inside and also some other points near to the origin are inside that sphere, so s-electrons can be inside the nuclear volume. The two charge distributions, the positive one and the negative one, they overlap, which is important, because we know, we realized, this condition of the nuclear coordinates being smaller than the electron coordinates is not fulfilled anymore, if this is possible. And therefore our multipole expansion will have overlap corrections. So the monopole shift, the first order monopole shift, is the leading correction to the monopole term, due to this electron penetration in the nucleus. And that was illustrated on this slide here, where you had the, for the hydrogen, the radial part of the wave function, for the 2s-electron, in red, and there you see that this non-zero at zero, so the s-electrons in general have this property of being present at the position of the nucleus, whereas the hydrogen 2p-electron, there the blue curve starts at zero, so 2p-electron will not be inside the nucleus, at least not in a non-relativistic picture. In a relativistic picture there is a small probability for p1-half electrons to be inside the nucleus, but the dominant contribution is always from the s-electrons. These are the same picture again, so for the 2s and 2p at the lower part of the slide, the radial part of the wave function, and here for the 1s-electron the probability density distribution. There was a question about this, somebody wondered, well I looked at the Orbitron website, and that was mentioned in this video, to the 2s and 2p orbitals, and I cannot match them to what is shown here on the slide, so what is going on there? Well I looked at the Orbitron too, and first for the 1s-electron, so probably the confusion comes from the fact that you have here at the top of the screen in the Orbitron, you have different features that you can plot, and one of them is the radial distribution function, so the one that peaks at non-zero distances, so that's this picture here for the 1s-electron, and another option is the wave function, so here this one is the radial part of the wave function for the 1s-electron in hydrogen. This one here was for the 2s-electron, so we cannot yet completely compare, and if you would square this function, then you would have the, and I go back a bit, and then you would have here the radial probability density, so for the 1s-electron, this one here is the square of this one. If you would do this for the 2s-electron, then the radial part of the wave function would be this one here, so non-zero at zero, and then it goes negative and goes asymptotically to the horizontal axis, well that's this red line here, that's exactly the same, and if you take the radial probability distribution, then you would have this feature, so a bit more complex than for the 1s-electron, so your Bohr radius would now be here. So I think what you see in the video and what is on the Orbitron is perfectly in agreement with each other, but you have to look at the correct functions, so I hope that this answers that question. If not, please ask again and try to be more specific where your problem is. Still nothing in the chat, so I'm a little bit worried about that, but well let's pretend that everything is all right and go on. Second task about the monopole shift, we look at the mathematical expression for that monopole shift that is repeated here on the slide, and that has the dimensions of an energy and that energy is made of a charge multiplied by a potential, what you have in brackets is the potential, and in the picture that we are thinking, we look from the position of the nucleus, we look at the fields generated by the electron distribution, the multiple fields generated by the electron distribution, and we multiply these with the multiple moments of the nucleus, so what we have here is the monopole moment of the nucleus, the ez, multiplied by the monopole field, which is the potential, the electric potential of the electron cloud at the nucleus. That would be the story in a pure multipole expansion without overlap. With this overlap correction, the one you see here on the slide, there you still have this feature of a charge multiplied by a potential, but the potential has the peculiar property that it is not dependent on the electron cloud alone anymore. In the monopole term, it was the potential generated at the position of the nucleus by the electron cloud, here it has a property of the electron cloud, namely the probability to find the electrons at the origin of the axis system, but multiplied by a property of the nucleus, the mean square radius of the nucleus. That's not familiar, not intuitive, but it is what happens in nature. And now I asked you, if we take that to the extreme, if we would take a nucleus that is very large, that has a radius that grows to infinity, then this term would also become infinite, and that doesn't feel right. You have a small correction and it can become infinitely large. What stops us from making this reasoning? Why is it not possible or where would our formalism break down if we try to make the nucleus infinitely large? Let's look at a few of your answers. Somebody said, I would think that the distribution term, so the charge distribution role in the nucleus, that this would prevent this mean square radius from becoming infinitely large. And indeed it does, and the fact that the charge distribution inside the nucleus is what it is, that dictates the mean square radius of the nucleus. So yes, that's correct what has been said there, but that's not what we are searching. We were searching, take it as a given that the nucleus is large, where would the formalism with its overlap contributions break down? What would prevent us there from making unphysical conclusions? Another answer was, well, if you would have an infinitely large nucleus, you would need a different theoretical framework. Exactly, but the question is then, how would that framework look like? What is wrong with our present reasoning that would forbid this? Other people gave an answer that was inspired by nuclear physics, giving nuclear physics reasons why a nucleus cannot become large. Totally correct, but we were again taking as a given that the nucleus would be large and we want to see where the formalism breaks down. Whether that is possible or not, that the nucleus can be large, that is not the issue here. There were several correct answers and two examples of this. One of you wrote the multipole expansion works because the size of the nucleus is really small. Indeed, that was a convergence criterion for the multipole expansion. If the nucleus would be infinitely large, then the expansion would not be converging. Indeed, we make a multipole expansion, that is always possible, sometimes with overlap, sometimes without overlap, depending on how your charge distributions are, but a multipole expansion is always possible, only it converges quickly if the nucleus is small. If you make the nucleus infinitely large, then no way that you can stop after the first multipole term or after the first overlap correction. You need everything in that case. So we should not worry about nuclei that are infinitely large and that would kill the physical interpretation of our monopole shift term because our formalism had started from the assumption that nuclei would be very small and our formalism only works if that assumption is true. Another way of formulating this, the same answer but formulated by someone else, we had deduced that the monopole shift was, in perturbation theory language, this matrix element here, evaluating a small perturbation operator that allowed electrons to be inside the nucleus, but if that nucleus would be very very large, then that perturbation would not be a small perturbation anymore, so it would not be allowed to apply first order perturbation theory. The formula for the monopole shift was a result from first order perturbation theory and assumed that this was the leading correction, that you could forget all higher orders of perturbation, that is only justified if the perturbation itself is small. So we have reasoned about this first order monopole shift, this leading correction to the monopole term. There are situations, however, where that first order correction is not sufficient and the clearest example for this is if you go to muonic atoms, atoms where the negative electron is replaced by an equally negative muon, but the muon has a mass that is much larger than the one of an electron, so therefore the orbitals of the muon will have smaller dimensions. And I ask you to think in the simple Bohr picture and find out what the dimensions of a 1s muon orbital would be. Many of you found that, it's just filling out some numbers in a straightforward equation and you find a radius that is much much smaller, that is two orders of magnitude smaller than the Bohr radius of the 1s electron. So muon orbitals are much closer to the nucleus and therefore the relative amount of muon penetration inside the nuclear volume will be much larger than for electrons. The overlap is much stronger, that's another way of saying this which means that not only this first order monopole shift will be large, but even that the first order monopole shift will not be sufficient. You will need to include the second order monopole shift in order to describe the energy of a muonic atom in sufficient detail. Some people thought that this was an exercise that was too simple. The exercise itself is simple, I agree with that, but I hope that by having done this once that you realize forever how small the orbit of a muon is in a muonic atom and that there are consequences for the and that there are consequences for the overlap contributions. So muonic atoms are an example where that second order monopole shift is relevant. So you see in this way, in this module, we have covered already this black box and this black box. There was a comment by somebody that I said in the video muonic atoms cannot be made anymore today and somebody said yeah but I have seen a thesis, a master thesis topic where muonic atoms play the role and yeah that's true. That video is a few years old and it is a feature of the past decade that muonic atoms are being studied again and facilities that were there in the 60s that have disappeared for quite some time, well such facilities are now active again, rebuilt with modern technology and nowadays people can make muonic atoms again and they are actively studied. At PSI in Switzerland there are facilities for doing exactly this. Not only muonic atoms, so let me make a small sidestep here and mention that there are also other exotic atoms possible nowadays, for instance anti-hydrogen. And the simple hydrogen atom but completely made out of antimatter. You have an antiproton in the nucleus, a negative charge and you have an anti-electron, a positron that is orbiting this, a positive charge and this is apart from the swap of the charges, a complete analog, a system that is completely analogous to the hydrogen atom, only made from antimatter. And you can, once you can create this and keep it isolated from real matter for sufficiently long, you can do measurements on this and well this is a very active field nowadays. I show a link to a paper that discusses some of these things if you are interested in this.

So you could study the hyperfine levels of an antimatter hydrogen atom and examine to what extent these are different or not from the hyperfine levels of a real hydrogen atom. Okay, no questions in the chat. Then we move to the second part of this module, the monopole shift illustrated by a toy model and I saw in the comments that several people liked this idea. I can understand that, I like this toy model very much myself. So what was the concept? We start here at the left hand side with a totally classic model for a hydrogen atom, well not necessarily a hydrogen atom, for some atom. We have an electron cloud that consists out of two negative point charges that are fixed in space and we have a nucleus that consists of two positive point charges that are connected by a rigid rod and as we think from the point of view of the nucleus, so we assume that the electrons are frozen in space and the nucleus can rotate, it cannot move but it can rotate, it can rotate freely around a central point here where the laser pointer is. And we wonder what is the total energy, electrostatic energy of this system as a function of the angle theta which gives the orientation of the nucleus. Only the angle theta because if you would let the nucleus rotate over an angle phi in the xy plane then that would give always the same energy, it's only theta that determines the electrostatic energy. And the result of this was this orange curve, so the energy is smallest when the nucleus has an angle theta 0 or 180 degrees, so when the nucleus is vertical and the energy is maximal when the energy is maximal when the nucleus is in the xy plane. And now we introduced overlap, we allow some charge to be inside the nucleus, so we put an extra negative charge here and we call this minus epsilon and epsilon is assumed to be much smaller than e which is the charge of these particles in the electron cloud. Because this is such a simple model we can calculate the exact electrostatic energy of this one and that's the black curve here and you see that this energy is lower, is more negative, is shifted downwards compared to the magenta line. What was the question I asked about this? Well we had seen that this first order monopole shift, that this gives always a positive contribution to the total energy. We could deduce that everything that appears here always leads to a total positive shift of the energy. Whereas, so here we saw it is a negative shift, we can even calculate this value, so this is the difference in energy between toy model A and toy model 0, the starting case, and that's a negative correction. So what is going on here? We have a solid derivation with the multipole expansion that shows that our monopole shift must be always positive and now we have the easiest example possible. We calculate this and we find it's negative. What is going on? Let's see. Somebody reasoned by adding a negative charge inside the nucleus, the total, the effective charge of that nucleus will become smaller and therefore the energy, the electrostatic energy of that entire atom must have been reduced. Yes, I agree with that, but that was not our question. We tried to ask why is that reduction not in agreement with the increase that we have derived for the general monopole shift. Somebody else tried to play with this and said, well, the only way I can see to make the energy rising is to put a positive charge inside the nucleus. Yeah, okay, sure, that would work, but this charge comes from electrons. We want to model how electrons are penetrating the nucleus. Electrons are penetrating the nucleus, so it must be a negative charge, no way around. Several people suggested the way how that toy model A is constructed is a bit deviating from what happens in real atoms, because in real atoms if part of the electron cloud is inside the nucleus, it means you would have to take away a bit of these two external charges and put that inside the nucleus. So you would have minus epsilon inside the nucleus and these two charges of minus e and minus e, they would be minus e plus epsilon over two. That's a good idea. I can understand why you think in this direction. However, if you try this, if you work out how large that effect would be, then you would see that subtracting this small charge from these two external charges, it will modify your expression, but to a very small amount. I was happy to see that one of you actually tried this, made some simulations and indeed came to the conclusion, no, it doesn't help, and the conclusion of the answer was probably I did something wrong, no, no. You should have found that this doesn't help, that was the correct answer to this hypothesis. So what is really going on then? The clue is that this negative contribution that we have calculated, that was the exact answer. Our toy model is so simple that we didn't care making a multipole expansion. We immediately found the exact answer and the exact answer, if we would have made a multipole expansion, that would have meant we take the monopole term. Now we realize there is overlap, so we have a first order monopole shift, we have a second order monopole shift, and so on up to infinity, but what is the exact answer? That's the sum of all these infinite number of terms. So we are talking about two different things. It's always true, even for this system, that the first order monopole shift will be a positive contribution, and for realistic atoms, there it stops, there is nothing more. But here in this simple system, if we calculate the exact result, so this is the exact result, so the sum of all of this, we didn't say anything about the second order monopole shift. Can this be positive or negative? We never discussed that, and apparently the total effect of all these further terms will be such that this positive contribution is turned into an overall negative one. If you would really try this, you would find that toy model A is a nasty beast for a multipole expansion. It's straightforward to find the exact answer, but treating this simple system with a multipole expansion, that leads to a lot of difficulties. You would find that the first order monopole shift is indeed positive, but even infinite, so this term is plus infinity. The second order multipole shift will be minus infinity, and this will alternate, plus infinity, minus infinity, plus infinity, minus infinity, and this an infinite amount of times. And the total sum of all these infinite contributions would be this finite value. So it's quite an involved expansion with singularities, where you have to apply the proper limits, and then you would find this, which, well, then you would find the same as what you find by making a simple electrostatic exercise. Which is a realization that I hope gives some extra insight in the nature of this first order correction. You find it in classical systems, only the behavior is a bit different, because the underlying assumptions, what is small, what is large, are not exactly the same. Looking back at the confidence questions, I can explain what the first order monopole shift is. I hope that by now you have a good view on this box. I can explain what the second order monopole shift is, so if the assumptions are not fulfilled to make this the leading term, or to make this the only term, if you have non-negligible contributions from other ones, then you need this second order monopole shift, think muonic atoms. There is also an overlap correction to the higher order multiple moments, so the first order quadrupole shifts. At this point, we didn't discuss the quadrupole term yet, so by now I don't expect that you have an intuitive picture about how this quadrupole shift looks like, but the concept that also the quadrupole term has an overlap correction, the concept, I hope that that at least is clear by now. Another one was, I can explain the meaning of the isotope shift and the isomer shifts, and I can point out how these are experimental manifestations of the monopole shifts. There are quite some people here who hesitate, and therefore let me summarize the answer to this statement. I think the statement is perhaps a bit more complex than the answer to the statement, and probably you will realize after the answer, oh yeah, sure, I get this. What is the isotope shift? That's in terms of our toy models, that would be two different energies for two different toy models that have the same electron distribution, but a different mean square radius of the nucleus, so a different length of the dumbbell, and in the case of the isotope shift, where do these differences in mean square radius come from? You have two isotopes of the same element, so same number of protons but different number of neutrons, and these do not necessarily lead to the same radius. So if the radius is different, even in the toy model, the energies are different, and that is if you measure energy levels of different isotopes, atomic energy levels that will be sufficient, you will find differences. This is the isotope shift, and that is an experimental manifestation of that first order correction term, that monopole shift. Another way how you can see this monopole shift in action, is by taking different states of the same nucleus, so you take one nucleus, one isotope, and you excite that nucleus, so you bring the neutrons and protons to higher orbitals, that too will have an impact on the mean square radius. These different excited states, these different isomers, will not necessarily have the same mean square radius. So again, in terms of the toy models, you will have two nuclei with different lengths, so the energy levels will be different. If you would measure the atomic energy levels with nuclei in different excited states, these energy levels would be different, and that is a manifestation of the monopole shift via nuclear effects, so via the isomer shift. So the general name of these two is the monopole shift, while the isotope shift and the isomer shift are two different ways to realize this. And that is what is summarized then in our very important picture number one, so we have positive shifts of these fine structure levels due to the size of the nucleus, either by having different isotopes or by different isomers. There was a question related to this, so somebody said, if I think in nuclear physics language, in the shell model, where the proton states and the neutron states are filled independently of each other, I don't get why the mean square radius would depend on the number of neutrons, because in the shell model the neutrons are treated separately from the protons, so as long as the number of protons remains the same, maybe the mass distribution within the nucleus changes, but the charge distribution, which is due to the protons, wouldn't that stay the same? So how can you have different isomers, or different isotopes, sorry, different isotopes with different mean square radii? Well, I'm not a nuclear physicist, so I cannot explain the answer to you in the details of the shell model, but at least I can look at some experimental data, and I quote here a paper from 20 years ago, where you have an extensive tabulation of mean square radii, and I take one part of the paper, the zirconium isotopes, so many different isotopes of the element zirconium, with always the mean square radius, and you see that this number changes quite a bit, so it's at least an experimental observation that the mean square radius of the charge distribution depends on the number of neutrons. My intuitive picture for this is that neutrons and protons, they do interact with each other via the strong force, and if you have a collection of protons with some distribution in space, so with a mean square radius, if you add extra neutrons to that, you will disturb the distributions of the protons, maybe not their orbitals in terms of the shell model, the name of their orbitals, and whether these orbitals are filled or not, these can be the same, but the spatial distribution, because the neutrons do interact with the protons via the strong force, the spatial distribution is not guaranteed to stay the same, and that is what you see here in this experimental table. I'm sure that this answer will not be 100% satisfying for somebody who searches really for the nuclear physics underpinning of this, but I suggest that you talk to the people in your environment who do know more details about nuclear physics, and if you find a really precise answer in nuclear physics language, feel free to let me know, and maybe I can include this in the discussions for next years. Okay, we are almost done. I finish with a few slides that I will just flash because of the references in looking at where monopole shifts appear in papers that have been published in the past few years, and I have just a few of them. So a nuclear physics paper from 2022, where laser spectroscopy was done, and I take one section of the paper where they, yeah, that could have come out of the text of this course, they just describe what the monopole shift is that they observe in their experiments. So it's a property that matters, it's a property that also in measurements that are being done today is playing its role, that's the message of showing this paper. Still things from the same paper, so numerical values for the isotope shifts, explanations why this happens, another paper from molecular physics where they take a single atom, dysprosium, and they measure energy levels of that dysprosium atom, and there too they meet the isotope shift, they give a discussion in the text of the paper what the isotope shift is, why it appears, they give experimental data where they can find numerical values for different isotope shifts in this dysprosium atom. So also here it's a property that is being actively measured today, that it's not some theoretical detail that you once learn about and you will never meet it again, that's what I wanted to illustrate with these two pictures. And by this we have reached the end of this webinar, still no questions in the chat, so I'm still a bit worried if somebody tried to put a question in the chat and this turned out and this turned out not to be possible for some reason, then please let me know via the I have a technical problem forum for instance, then I can try to find out what was the reason. But if you were just happy and felt no need to put questions in the chat, that's equally fine of course. I will wait one more minute to see if a final question appears, because due to this small time delay I'm talking maybe 20 seconds before you really hear it. So let's wait one more minute. Aha, okay, I saw the problem is at my side, the chat was, I showed a different version of the chat window somehow, must have touched a button without knowing. So I see the confirmations of the audio from the beginning. Sorry, I will pay attention to this configuration the next time. Okay, so everything is fine then and you can start the module now on the magnetic hyperfine interaction and we will give feedback on this in one week from now, same place, same time. Bye-bye.