webinar 5: the magnetic hyperfine interaction

(automated transcription)

This is the feedback webinar on the Magnetic Hyperfine Interaction, the second of our Physically Observable Hyperfine Interactions. This week there is no administrative information except for the fact that I noticed yesterday evening that the introduction page in the course site has not the proper content. So for the Magnetic Hyperfine Interaction it starts with the same introduction as for the monopole shift, which is obviously not right. I couldn't update it yet, but I will do so soon and hopefully this didn't bring any confusion among you. In technical terms this Magnetic Hyperfine Interaction is the dipole term in a multipole expansion, but not the multipole expansion of two charge distributions as we had with the monopole shift. No it's the multipole expansion of two current distributions, the currents that are inside the nuclei and the currents that are in the electron cloud. These two current distributions you can multipole expand them, the monopole term is zero, the dipole term is the first non-zero term and that's the Magnetic Hyperfine Interaction. And it's this dipole term that if you apply perturbation theory will give rise to these hyperfine split levels at the right hand side of the picture. Here it will be an interaction between two vector properties as we are in a dipole term, so we will have a vector associated to the electron cloud, that's the magnetic hyperfine field and a vector associated to the nucleus, that's the nuclear magnetic moment. We have discussed this in the course in two sections, two main sections, the Magnetic Hyperfine Interaction for free atoms and the Magnetic Hyperfine Interactions for crystals, for solids and the reason is that the mathematical description in both cases is a bit different. You can benefit from features of free atoms that can give rise to a shorter mathematical description and it's even particularly easy for those of you who have studied in detail spin-orbit coupling before because the formalism of spin-orbit coupling and the formalism for the Magnetic Hyperfine Interaction in free atoms is identical, very very similar. So therefore let me very quickly go through the reasoning of spin-orbit coupling once again and then duplicate that for the Magnetic Hyperfine Interaction. So in spin-orbit coupling we have one vector property that describes the orbit of the electron and that's the angular momentum vector of the electron orbit, here symbolized in a very classical way where the electron is on an orbit like a planet and the vector, the orbital angular momentum is a vector that is perpendicular to the plane of that orbit. So in a way the orientation of that angular momentum vector gives you some information about the orbit, you know the plane of the orbit. If the orbit would be in a different plane the angular momentum vector would be pointing in a different direction. That's one vector that plays a role, that orbital angular momentum vector. The second vector is the spin angular momentum vector of the electron and we can wonder how is that spin angular momentum vector oriented with respect to the orbital angular momentum vector? And this mutual orientation for atoms is given by the three Hund's rules. If you apply the three Hund's rules at the end you know how S is oriented with respect to L. And that is what gives rise to the fine structure levels. For different mutual orientations of S versus L you have different fine structure levels. Now we move to the hyperfine levels and ok, before doing so, indeed I asked you a question about this. Look at this picture, the picture was missing in the first few days of the week, there was a broken link and somebody reported it in the forum, since then it has been fixed. So this was the picture that some of you may not have seen and the question here was find the levels where the L and S are parallel and pointing in the same direction, in opposite directions or more perpendicular to each other. And the correct answer would be if J is maximal then L and S are added, you get maximal value so then they are pointing in the same direction. If the value of J is minimal, so in this case zero, then they are pointing in opposite directions because you have L minus S and the case in between, the half way between same direction and opposite direction is perpendicular. And now we move this reasoning towards the hyperfine levels where we don't have L-S coupling but now we study I-J coupling, I the spin of the nucleus and J the total angular momentum of the electron cloud, so the one that is

made from L and S. And I will take here as an example an atom where the electron cloud is in a state that can be described by J equals 2, just to have a number. That would look on the very important picture number 1 something like this, we have somewhere a level in a particular fine structure and now we consider this hyperfine splitting and we look at the ground state and the lowest state in this diagram where J equals 2. And here we can wonder, can we understand that the electron cloud that is in state J equals 2, that that can produce a magnetic field at the position of the nucleus. And I write there hint, this J is made from an orbital and a spin part. Let me continue the classical reasoning, if the electron is in an orbit then it represents a current, a charge that orbits is a current and a current loop, there you know from classical electromagnetism that it generates a magnetic field. Also at the position of the nucleus there will be a magnetic field due to that current. That is the orbital contribution to the magnetic hyperfine field. Magnetic hyperfine field is the field generated by the electron cloud at the position of the nucleus and here we see that it has an orbital contribution and by the orbital motion of the electron there is a contribution to that hyperfine field. The electron has also a spin which can classically be represented by a bar magnet and even if the electron would not be moving, if it would be at rest, if that is somehow possible, then by the mere presence of that bar magnet there will also be a contribution to the magnetic field at the position of the nucleus. And that's the spin dipolar contribution to the hyperfine field. So that was about the J, the electron contribution, now the I, the nucleus, the nucleus has spin as well, so the nucleus is also a bar magnet and that nucleus, that bar magnet is in the hyperfine field, this orbital and spin hyperfine field at the position of the nucleus. What does a bar magnet or a magnetic moment in a magnetic field does? It tries to orient itself towards the position that is energetically most favourable and how is that described? In classical electromagnetism the interaction energy between a moment and a field is minus mu dot B, so it depends on the angle that the magnetic moment vector makes with the magnetic field vector. You could make a graph of this, if the moment and the field are parallel and point in the same direction, then due to that minus sign in the expression, the energy is lowest. So I have here on this vertical axis, switch on the pointer, on this vertical axis we have here the lowest possible value of the energy and here I show that same energy, but now on an energy axis versus this angle theta in degrees. So if you turn the magnetic moment, then the energy increases, perpendicular then it is half way, it increases even more and if the magnetic moment is anti-parallel to the field then you reach the maximal energy. Very classical, I could describe this as a spread of energy values, the black box gives the range of all energies that are covered by one possible orientation of moment versus field and in a classical situation all of these energies can be reached. Now I put this classical picture on my fine structure level, so I have a fine structure level in which the electron cloud is in a state J equals 2, we have seen this gives rise to a magnetic hyperfine field, now my nucleus, depending on how the nucleus is oriented with respect to the hyperfine field, you can get any of the energies in that black area, but we have here a quantum system and not a classical system, so not all of these energies are allowed, not all of these orientations are allowed. If my nucleus would have a spin 1, as an example, then instead of all possible orientations all possible angles theta, only 3 possible orientations are possible, corresponding to the 3 quantum numbers mI being minus 1, 0 or plus 1. I now see that I have written mJ there, I should update that. So out of the full black box with an infinite number of values, only 3 values are allowed and these are the hyperfine levels. Every hyperfine level corresponds to a particular orientation of the nucleus with respect to the magnetic hyperfine field generated by the electron cloud. So in exactly the same way as L and S give rise to a J in the spin-orbit coupling case, we have here an I and a J, the nuclear spin and the electron angular momentum, that give rise to a new quantum number f, which describes the relative orientation of the I versus J. And a different value of f means that you have a different of the allowed energy levels. So on an example where we have I 3 halves and J 3 halves, the possible values of f will be 0, 1, 2 and 3. In exactly the same way as you get all possible J values from L and S, you get all possible f values from I and J. You go from the minimal value, which is the absolute value of I minus J, that would be 0 here, to I plus J, 3 halves plus 3 halves is 3. So 0 and 3 are the minimal and maximal values and then all integer of all steps of 1 in between are also

allowed. So 0, 1, 2 and 3, 4 levels. If you look in detail at the energy spectrum of a free atom, where the electron cloud is in a state described by J 3 halves and where the nucleus has spin 3 halves, then there will be 4 possible mutual orientations of I versus J, so you will see these 4 different hyperfine levels. So on the confidence question I can explain how the coupling of angular momenta is related to magnetic hyperfine interactions in free atoms, that's basically the story I told, I repeated just now. A related question, I can explain what Landé's interval rule is and how it relates to the magnetic hyperfine interaction. Some of you may have seen Landé's interval rule in the context of spin-orbit coupling. It tells that the ratio between subsequent fine structure transitions, that this ratio is identical to J versus J minus 1 for each of these levels, because the formalism is the same, exactly the same ratio applies to the magnetic hyperfine structure, but then it will be F over F minus 1. A question where there is some tale of uncertainty, starting from the classical expression for the magnetic dipole energy, I can construct a Hamiltonian in a form for which I know the eigenvalues. What did I mean by that? The key point is starting from that classical expression, and I've just repeated what that classical expression is, minus mu dot B. You will remember the slide in the video that is shown here, where it was sketched how starting from that classical expression, now with operators, if you fill out the general descriptions of these operators, and then play the trick where you replace I dot J by this sum of squares, these are operators for which you do now the eigenvalues. These are squares of angular momenta operators, that is something where we know the eigenvalues. We do not know the eigenvalues for I dot J, but in this different formulation with the squares of angular momenta we do know them. So in this way we have a hyperfine Hamiltonian that has for every part of it known eigenvalues. So we can completely go to the finish and have quantitative expressions for the hyperfine levels in free atoms. And because we treat this as a perturbing Hamiltonian, we can get to these quantitative values via the formalism of perturbation theory, and the results, it will be degenerate first order perturbation theory, we will have a matrix, and if we diagonalise the matrix, the diagonal elements, these will give the position of the hyperfine levels like here. Now in order to get a little bit familiar with the calculations in that formalism, I ask you to calculate one of these matrix elements in that degenerate first order perturbation theory story, namely the 1,1 element for the perturbing Hamiltonian Hjj. And do that for a nucleus that has spin 3 halves and total angular momentum 3 halves. So if you don't remember where it came from, this is the C11, this element here, which is a particular version of this expression. And I show you one of your answers to point to the fact that in this answer there is an h bar squared remaining. And that is why these simple exercises are helpful. By having to write it down, apparently some people hesitate, should there be an h bar squared or not. Look back to the derivation, indeed when we start from operators, and if we apply the operator on an angular momentum eigenstate, we get an h bar squared, that's this one here, but in the prefactor there was a division by h bar squared, so this cancels out and in the matrix element where we have already the eigenvalues and not the operators anymore, the h bar squared has gone. So in your C11 there should be no h bar squared. And these are a few answers that were correct, twice the same reasoning, twice the same answer, you find C11 is minus 11 over 2, so that's the correct answer, so that's this C here and that needs to be multiplied by minus one half times the hyperfine structure constant. So this, just to emphasize that once again, these values, these C values, these are the matrix elements in a first order perturbation treatment, degenerate. So well, okay let me repeat also that part, without the perturbing Hamiltonian, without the magnetic hyperfine interaction, it does not matter what is the mutual orientation of I and J, the magnetic moment of the nucleus can point in any direction that will have no effect on the total energy of the system. So I and J, or the mutual orientation of I and J, which means F, F does not matter for the total energy, so the energy is degenerate with respect to F, that is why we will end up in degenerate perturbation theory. Now we switch on the magnetic hyperfine interaction, we introduce the perturbing Hamiltonian and we have to evaluate the old eigenstates, these different F states in the perturbing Hamiltonian, so we have to make the matrix of all possibilities, all possible states F from, okay in this case from zero to three, because it was still with the example I three halves, J three halves, so we have a four by four matrix and we find already by construction

that this will be a diagonal matrix here, so we will be lucky, there is no diagonalization needed, no explicit diagonalization needed, we have on the diagonal the values that directly give the contribution to the hyperfine splitting. And there was a specific question on this one, that's why I elaborated a bit on it, somebody wondered is that matrix in the first order perturbation always a four by four matrix or was it specific for our example where we have four values of F, so yes definitely this was specific for the example, if you would have seventeen values of F then that would be a seventeen by seventeen matrix, it was just to be able to write a matrix there, it's not meant for the general case. Okay let me see whether there is a question on the chat, is the Fermi contact contribution not a part of the J contribution, yes it is, but in my classical story that I told so far, the Fermi contact contribution has no place in the classical story, we will come back to it at the end of this webinar when we will be dealing with the overlap contribution, then every piece of the puzzle in the framework will be put in the proper place. And then another question, if you took the state F equals one, you would have, I'm not sure whether I understand that question completely, so I will just read it aloud but it's difficult with a bit of symbols, if you took the state F equals one, you would have that one is three halves plus J, so J is, okay, so if you would take, we have our four states for F from zero to three, I told that the zero is I minus J, absolute value, the three is I plus J, no need for an absolute value because that will always be positive, and then integer steps in between, or steps of one in between, and the question here is, let's pick one of these intermediate values and let's figure out which I and J I need to find that one, so then you find some impossible value of J, so the point is you cannot write that as simply for these intermediate values, you have very short expressions for the two extremes, if you would want to express the intermediate values, that is possible, if you have seen a course on the quantum treatment for angular momenta then something as Clebsch-Gordan coefficients might sound familiar to you, you would need an expression with Clebsch-Gordan coefficients for these intermediate ones, the simple algebra will not work there, and that reminds me to another comment that one of you wrote in one of the tasks, where somebody gave a very detailed algebraic mathematical discussion of the role of angular momenta, and was complaining that these classical pictures were a bit too simple and were confusing, and that's indeed a choice one has to make here, either you describe this in a conceptual way, where there is still some relation to what you can classically imagine, and that's the path I have taken for this course, I try to be correct, but pointing nevertheless to similarities with the classical case, or you could take the alternative approach and go fully in the quantum formalism for angular momentum, where things get very algebraic and you have no visual representations anymore, and you can just believe your final numbers, that's of course the most correct way, but that would require a different type of course, then we would need to spend many hours on that formalism first, and that's not the goal, we want to have reasonably correct mental pictures of hyperfine interactions, so no Clebsch-Gordan coefficients here. The next task was a situation where we have a nucleus with nuclear spin I1 that is known, nuclear magnetic moment mu1 that is known, and you have somehow the possibility to measure the hyperfine splitting in a free atom of a system where that nucleus is contained. And now you get a different isotope of the same element, with again a nuclear spin I2 that you know, but a magnetic moment mu2 that you do not know. With the tools you have at hand, how would you proceed to determine experimentally that magnetic moment mu2? Many of you have described a correct reasoning, so I will go through one reasoning step by step. So I have written at the right-hand side the two key expressions, the hyperfine structure constant for the first system, the first isotope A1, is the first magnetic moment times the hyperfine field in that system divided by the nuclear spin and the electron angular momentum, and the same for the second system. So in our first system we measure this hyperfine structure constant, we measure the difference between two hyperfine structure levels basically. Then we see that this depends on an expression where we know some things, we know the nuclear magnetic moment of the first isotope, so we know the mu1, we know the I1, what we don't know is the value for the hyperfine field and the state of the electron cloud of the atom in which that nucleus is. So mu1 over I1 we do know, B1 over J1 we do not know. Then we, okay sorry, but yeah, no, I should say that more precisely, we do know the ratio B1 over J1

because we can get from this expression, but we do not know B1 and J1 individually. Then we move to the second isotope where we can write the same expression, but because we have now the same atom where only the nucleus is replaced by a different isotope, that means everything, basically everything remains the same apart from the hyperfine structure. We have a nucleus with the same charge, it's the same isotope, so up to the magnetic moment of the nucleus, the hyperfine term, up to that point everything is the same. So if we have the same system, the same nucleus, the same atom, the hyperfine field at that nucleus should also be the same. So B1 is the same as in the previous case and J1, the state of the electron cloud, is also the same. So that ratio B1 over J1 we can just use it in our second expression. We have determined it from the first expression, we put it in the second expression and then we measure again the hyperfine splitting, the A2 now, and because we know the spin of that second nucleus that was given, then the only unknown is the magnetic moment. This is a kind of reasoning that is often used in the practical use of hyperfine interactions. You have a system where you know everything about the nucleus and then you measure a similar system where you go to a different isotope where some things are unknown and by comparing the measurements in the two systems you can find out properties of the unknown nucleus. And that's a very important message. So let that sink in how strange and useful this is, because what we have done here is two measurements on energy levels of atoms. We look at the total energy structure of an atom, we measure it with very high resolutions such that we are able to resolve the hyperfine levels, we measure these hyperfine levels in two different atoms that contain a nucleus of two different isotopes and the result of these measurements is that we learn what is the magnetic moment of an unknown isotope. So we have measured a nuclear property by doing two atomic measurements. That's a way how to get to nuclear information. And you don't need nuclear physics so to say, you don't have to measure directly on the nucleus, you measure on the full system on the atom and still you get the nuclear property. That's a very useful application of hyperfine interactions. Okay, with this we finish the topic for free atoms. Let me go back to the chat, nothing new so far. And we continue with the magnetic hyperfine interaction in solids, in crystals. And here I asked you for three different systems, a particular isotope of cadmium, a free electron and a free neutron to determine with the known information of these systems to determine the g-factor. That sounds like a silly exercise. I think two or three even commented that this was not a particularly useful exercise. I will try to demonstrate why it is useful. And the reason is, well this is one of your answers. Somebody says you start from the formula that is given there at the top in purple, this expression, and then everything you have to do is just for the three cases fill out the numbers and there you are. Now you see in this answer it is described how to do that, but there are no final values. And I would really recommend to do it with the numbers that were given to get to the final values because that is where you will see whether you understand this expression or not. The devil is in the details here. Let's look at another answer where they start from the same expression. Here they do get numerical values, but these are not always right. And the reason is that expression where you start from. Well, I think I know where you got it. The people who started from that particular expression, they probably looked at this slide, where you have a relation between the nuclear magnetic moment operator and the nuclear angular momentum operator. And the proportionality factor between these two, that's this expression here. However, we don't work with operators if we look at experimental values of spins and magnetic moments. We work with their measured values. So first we have to wonder how does this relation look like between the measured properties. And, well, let's try to convert that. You have seen this slide as well. And this expression is repeated here or here. Well, how can we get the experimental value? The convention is you first go to the z-component. So you take the z-component of the magnetic moment operator, the z-component of the angular momentum operator. And now you evaluate these z-components in a state where the m-value is maximal. So the z evaluated in a state with maximal m, so where m equals i, that is the experimental value. This is the magnetic moment that you measure. And if I evaluate this expression, because we know the eigenvalues of Iz, you get this. So the experimental value mu is the g-factor times the nuclear magneton times I. And if I want to have the g-factor out of this, so that's mu over mu n times I, or if

you do this for a situation where no nuclei are involved, where it is about electron properties, then you don't have the nuclear magneton but the Bohr magneton. The expression is the same. So the expression we have to use, if we use tabulated experimental values, is this one here. And you see that this looks a bit different from the one between operators. For instance, this h-bar is not there. If you have an h-bar in your expression, then you're already sure that you made this mistake here. Also, if we have nuclear moments tabulated in nuclear magnetons, so often the tables will tell you the nuclear moment is that many times the nuclear magneton, then this number here is effectively some number times the nuclear magneton. And because you divide here by the nuclear magneton, well, these two disappear. And what is left is the nuclear magnetic moment in units of nuclear magneton, so that's the value you have in the tabulation, divided by the spin. So a very simple expression. If you work with tabulated values, it's the tabulated nuclear magnetic moment divided by the spin. That's the g-factor. And so if you apply this to this example for 111 cadmium, you get these numbers. So from this you can immediately see whether you got this right or not. And well, I noticed there was somebody who explicitly wrote, look, I have this expression, you see here the h-bar appearing, so that was wrong. But the point here is now, I do not know how to convert the units, because mu is given in nuclear magnetons in the database. Well, that's exactly why this, so 0.766 times the nuclear magneton, and here divided by the nuclear magneton, so this nuclear magneton disappears from the expression, the h-bar was there not as well, and you have just the tabulated value divided by the spin. And for the other cases, it's even simpler. So in the case of the electron, there you have the Bohr magneton. What is the electron magnetic moment? That's one Bohr magneton, that's how the Bohr magneton is defined. So in units of the Bohr magneton, in units of the Bohr magneton, the electron magnetic moment is 1 divided by the spin, which is 1.5, so 1 over 1.5, that's 2, so the g-factor for the free electron is 2, which is what you find here, and a similar reasoning for the neutron. So, I know, an exercise that may look dull, but it's important that you see the difference between working with operators and working with experimental values. And if you read a paper, if you want to interpret results in a paper, you will see these reported experimental values, and if you then take the operator expression to process these values, it will lead you nowhere. So I think this is maybe a bit dull, but still relevant. Yeah, this h-bar, okay. And questions in the chat. Does the Bohr magneton work for muons? Yes, muons are particles that are not nuclear, so with muons it's the Bohr magneton that is used. Aha, and okay, so you will have been the one who wrote that example, the nm is nuclear magneton, not nanometers. Okay, so that was the confusion. Okay, got it. Then, a kind of thought experiment. You have a carousel, a child, a Bohr magnet, an electrically charged ball, and a magnetometer, and you have to use these ingredients to recreate as many contributions to the magnetic hyperfine field as possible. Again, an exercise that some people find very interesting, and other people find, well, below their level. But it's meant to think about the actual meaning, the physical meaning of these different contributions. And I find it easier to think about this in a totally innocent context, than in a physics context, where you are sometimes used to formula about which you do not think anymore. So, by replacing this outside the physics context, you can look at it with more fresh eyes. That is the purpose of this exercise. And I show here a combination of two of your answers that shows what you got and where the problem is. So, yes, you can put the magnetometer in the middle of the carousel, at the place where the nucleus would be. So the magnetometer will measure the magnetic field at that position. So that will be the measurement of the hyperfine field. The child on the carousel with the charged ball, the child is the electron. If that child has the magnet in the hand, and the carousel does not move, so the child is at rest, then that magnet will produce a magnetic field in its environment, and that will reach up to the magnetometer. And the magnetometer will register, there is a magnetic field here, from the bar magnet that the child holds. So that's the spin contribution to the magnetic hyperfine field. If the child also has this electrically charged ball, and the carousel does not move, then that will have no effect on the magnetometer. But if the carousel starts turning around, so the charged ball is on a circular orbit, it is a current, then that current will create a magnetic field, and the magnetometer will see that. That's the orbital contribution to the magnetic hyperfine field. So many of you agree on that. The

blue part, that is where it gets difficult. I do not know how to simulate the contribution of the Fermi contact term. As in this case, the bar magnet would have to be inside the magnetometer. Okay, well, let's compare that to somebody who tried to simulate that. So in this answer you have in the different colors the three different aspects about which we agree. And then the last one, which is attempted to be the representation of the Fermi contact contribution. Let's assume that the child has again that charged ball, the carousel is spinning, and the child is wandering wherever he or she wants on the carousel, even inside the nucleus while holding that charged ball. So, I don't completely agree with it, but okay, you cannot now, because this thought experiment is meant to point out what is special about a Fermi contact contribution. You do have a classical counterpart for the orbital and the spin contribution. Do we have a classical counterpart for the contact term? Not really, at least not physically. Mathematically, how could we have a Fermi contact term mathematically in a classical context? It would happen when we would have, yeah, when, I'm thinking how to formulate that, which is difficult because there is no classical counterpart. We would need to have two magnetic monopoles, a magnetic south pole and a magnetic north pole. And inside that monopole there should be a current. Now, that's not possible because we cannot separate north and south poles. We have only magnetic dipoles, we do not have magnetic monopoles. But in the context of the nucleus, where this spin up and spin down, so where there is a kind of spin current at exactly one point at r equals zero. So, in our formalism, in the Fermi contact term, we have, we need the spin up versus spin down difference at r equals zero at one mathematical point. And at that same mathematical point, we have our nucleus. Well, the point is infinite in dimensions, so it is somehow inside this north and south pole of the nucleus. So this is a very peculiar situation where we can get inside the bar magnet, in between the north and the south pole. Which is, well, a kind of visual explanation why we can have a thing as the Fermi contact term in the nuclear context, but not in the classical context. Not because it is fundamentally forbidden in the classical context, but we have no way to get in between the north and the south pole of classical objects. In this nuclear situation, we can do that. That brings us to the last part, the overlap contribution. And here I asked you to imagine a perfectly spherical nucleus with a non-zero radius. And somehow the electron cloud is such that all electron charges stay outside that nucleus. Whether that is really possible or not, we don't care. We just assume that we have that situation. Will there be energy corrections due to the finite size of the nucleus? Now, we are still technically struggling in the website with these post-first forums. So now you can, for the time being, you see all the posts. And I know you will read a few of the answers before you post your own one. And so therefore there is sometimes little doubt about what the correct answer is. Now, in the years when this post-first forum was properly working, where everybody had to post an answer without being influenced by the others, then very often the answer is no. In this situation, there is no overlap of the two current distributions, so there will not be such an overlap contribution. Now, well, this year the typical answer looks like this. I have two examples of them. So the Bohr Weisskopf effect, it does not require electrons to be in the nucleus. Only a magnetic field created by the electron cloud should be in the nucleus. And that is perfectly possible without the electrons entering. So at this stage, yes, there can be an overlap contribution. There can be a Bohr Weisskopf contribution. This particular answer continues. Since the nucleus in this example is a perfect sphere, I imagine there would be no hyperfine anomaly term. Because the energy ratio would be the same as the nucleus in an external uniform field. And that's an interesting reasoning because that shows a little misunderstanding. We can have a hyperfine anomaly or an overlap contribution due to the fact that the field is non-uniform. But also because the magnetization in the nucleus is not uniform. So even if we have a homogeneous field, well, no, no, let me say this in the other way. Both aspects matter. We need a field that is not uniform, but also the magnetization distribution matters. Why will two different isotopes have a different hyperfine interaction? Because that hyperfine field that is not uniform interacts with the magnetization that is not uniformly distributed. And it's not because we have here in this hypothetical example a perfect sphere that the nuclear magnetization cannot be not uniformly distributed within that sphere. So you could have a hyperfine anomaly even with a nucleus that is perfectly spherical. So in this

answer, this is formulated in a more correct way. So yes, in our hypothetical example, there will still be a contribution Bohr-Weisskopf effect. The Fermi contact contribution is zero, since there are no electrons inside the nucleus. But the Bohr-Weisskopf effect, that will be present because the magnetic field generated by the electron cloud is still present within the nucleus. And you will be sensitive to the different ways in which the magnetic moment of the nucleus is distributed over the nuclear volume. So I've repeated there at the bottom of the slide the two aspects that play a role. You have a nuclear moment that is not distributed evenly over space, where space is the nuclear volume. And you have a hyperfine field, a magnetic field by in this case only the spin and orbital, that should be the orbital and the spin dipolar contributions. You have a magnetic hyperfine field that is not constant over the nuclear volume. And by the interplay of these two effects, you are, the hyperfine interaction is sensitive to the distribution of the nuclear moment. So this is another illustration how hyperfine interactions can be useful for nuclear physics. Because if you measure so precisely that you can detect the hyperfine anomaly, then you have an experimental access to how nuclear moments are distributed over the nuclear volume. And that can be compared with predictions from nuclear theory. One model for the nucleus may predict a different distribution than another model for the nucleus. And by comparing this with experiments, you can find out which of the two models is the more correct one. There was a question here, somebody asked, you say that this Fermi contact contribution is a first order correction for a dipole moment, but what about these other two, the orbital and the spin contribution, are they also part of the zeroth order correction? So I see a lot of confusions in this question. And the answer to that question is in the thing we started from. We are discussing here the dipole term in a multiple expansion of a current-current interaction where the monopole term was zero, the dipole term is the leading one. And in this slide in the video, it was said that in the pure multiple expansion, so without overlap corrections, in the pure multiple expansion, your dipole term looks like this and contains only the orbital and spin contributions to the hyperfine field. So if you have a hypothetical atom where the nucleus is a point, is really a point, but a point that has a magnetic moment, well then you will have as leading term in the multiple expansion only this. However, if you allow the two current distributions, the one from the nucleus and the one from the electron cloud to overlap, so if the condition are smaller than, always smaller than, are larger than, if that is not fulfilled for this current-current multiple expansion, then you will find two corrections. Two now, so your entire series, what we saw last time with the monopole shift, in the first-order monopole shift, the second-order monopole shift, well here you will have the first-order dipole shift. The Fermi contact contribution is actually a first-order dipole shift and higher order terms. And well, these consist now of two parts. You have this Fermi contact contribution, that depends only on what happens at the position r equals zero, so at really the center of charge of the nucleus at one mathematical point. So even if the nucleus really becomes a point, but a point with a magnetic moment, this part of the overlap contribution will not vanish. And then there is the field correction due to the Bohr Weisskopf effect, that requires the nucleus to have a certain volume, because you need to have the magnetic moment distributed over a volume in a particular way. So if you have two isotopes with two different ways of distributing that magnetic moment, then you will have two different Bohr Weisskopf fields. So this is how everything fits together. If I go back to this question, what is an overlap correction? What is a proper multipole term? So orbital and spin contributions are part of the normal dipole term, no overlap correction. And Fermi contact and Bohr Weisskopf, these are overlap corrections. And they are slightly different from the charge-charge case, because here we have a contribution that would not vanish if the nucleus has a zero radius. If you think about the monopole shift, charge-charge interaction, the monopole shift was dependent on the mean square radius of the nucleus, so it would vanish if you have a zero radius. So the monopole shift is basically at the same level as the Bohr Weisskopf effect. The special new thing for the current-current interaction is that you have this overlap contribution that does not vanish with a zero radius. And as you saw in the numerical examples in the video, it's often this contribution that is even dominant. So if you measure a magnetic hyperfine field, in many cases it will be overwhelmed by this term here. So the first order dipole shift can be a large one. And the

last task was to ponder these four statements and to indicate the ones that are meaningful statements. There were some interesting answers on that. Somebody indicated only this one, the hyperfine anomaly for the isotope gold-197 is 2%. And why was that taken? Well, different arguments to exclude the other ones. And the hyperfine anomaly for two different elements, like here, that is not defined. In the third case, I don't think you can have a hyperfine anomaly for an entire element. So if you say hyperfine anomaly just for platinum, that is not a meaningful statement either. And both these arguments are correct ones. The argument that is not correct, which is used to eliminate the second line, I don't think you can compare two different isotopes because the Bohr-Weisskopf effect has to do with the distribution of magnetic moments in the same nuclei. And that's a misunderstanding, because one nucleus will always have the same distribution of magnetic moments. You cannot say here you have one nucleus with one distribution of magnetic moments and now you take exactly the same nucleus that has a different distribution of magnetic moments. No, the identical nuclei are identical. But what you can say is you take one isotope with a distribution of magnetic moments and now you take a different isotope of the same element, so some neutrons are added or subtracted and that will have in general a different magnetic moment that is distributed differently. Another answer where two boxes were ticked. You can only talk about the hyperfine anomaly when comparing two different nuclei, because you compare the ratio of energies within identical externally applied magnetic fields and identical hyperfine fields. And the word that is wrong here is nuclei, so in the correct version it will be isotopes. And the second thing that is wrong here, the idea of comparing with a naked nucleus without any electrons inside an uniformly applied magnetic field, that was not meant as an actual measurement. That is a hypothetical step in the reasoning to point out why the hyperfine anomaly exists. But if you measure it in practice, you will not measure a naked nucleus in a uniformly applied field, you will always measure a particular isotope in a hyperfine field and compare the differences. I think the same confusion was also present here. The correct answer is to tick only the second line, that's the only line that is meaningful and the green argument there is correct. The hyperfine anomaly is the correction to the energy ratio when you compare two different isotopes. And then what was wrong in this answer had ticked also the first one, for the same reason as I mentioned on the previous slide, because you can compare a particular isotope with that same isotope without electrons in a uniform field. Now in principle you could do that, but in practice this is not done, this is not easily measurable. Then somebody ticked answer number one, for a reason that I don't completely understand right now. So it would be easier to specify the hyperfine anomaly of one specific isotope, probably because you are also thinking compare that case to the uniform external field, of which I just said this is in practice not doable. So the only correct one was this second line, and we have always specified a hyperfine anomaly between two different isotopes. If you see in a publication a hyperfine anomaly mentioned, then it is between two isotopes. The word hyperfine anomaly compares two isotopes. Okay, with this I've reached the end. Let me look at the chat. Does this connect with our toy model of a nucleus as a dumbbell? And I assume that this is about, that the question refers to the picture with the multipole expansion of the current-current distribution. So good that you asked this, and it allows me to emphasize that the toy model is explicitly for the charge-charge interaction. In the toy model you have two static charges, the electrons, and a dumbbell that can rotate and that contains charges for the nucleus, but we consider only the Coulomb interaction between these charges. There is no current involved. So the toy model is not applicable to the current-current interaction. Let me see if I would think here on the spot, would it be possible to construct a similar toy model for the current-current interaction? Then you would probably, maybe a variant of that double ring system that we discussed before, but now a double ring that has a current, that carries a current, and the nucleus, the dumbbell, would then not be a dumbbell, but a ring as well. And then you can classically calculate how the ring, the nuclear ring, which is represented by a magnetic moment, interacts with the magnetic field generated by these two current rings for the electrons, and then try to make a multipole expansion out of that. Yeah, which will not be easy mathematically because the multipole expansion for the current-current interaction contains vectors, or contains

a vector potential compared to the scalar potential of the static charged-charged interaction. It will be more involved, but I'm sure that technically it can be done. That would be a kind of elaborate project, if somebody wants to play with that, feel free to go ahead, that would be interesting, but not a simple task. Okay, I will wait for one more minute to see if there are additional questions on the chat. That does not seem to be the case, so I will stop here. And we'll see each other again next week for the electric field gradient, the quadrupole interaction. Bye-bye!