toy model for the monopole shift

(automated transcription)

You will understand by now that as well in the gravitational case, where two mass distributions are interacting, as in the electrical case, where two charge distributions are interacting, a monopole shift can appear. It is quite instructive to look at this monopole shift in a particular case that is so simple that we can do exact calculations, so a case where we don't need the multipole expansion anymore. So let's go back to our gravitational example, but let's consider it immediately as an electric example. So the red distribution you see there is a charge distribution, the blue distribution is also a charge distribution, the blue one is positive, the red one is negative. So the mental picture is that this is a classical version of an atom with an electron cloud and a nucleus. Let us simplify this charge distribution, these two charge distributions, in such a way that it becomes computable. And simplifying, you can take that quite literally, we simplify the electron charge distribution until we have two point charges, two elemental point charges, so one electron as a classical point particle, and these two point charges are fixed on the vertical axis. Also the nuclear charge distribution is simplified until you get a dumbbell of two positive elementary charges. The center of this dumbbell is fixed to the point that is halfway between the two electrons, and around this fixed point this dumbbell nucleus can rotate in three dimensions. You see the same picture here again, in order to introduce the symbols, so 2L is the length of the dumbbell, the angle theta gives the orientation of the dumbbell with respect to the z-axis, and the angle phi gives the orientation of the dumbbell in the x-y plane. For the electrons, the distance between the two electrons is 2D. You will readily believe me that it is an elementary exercise in electrostatics to calculate the Coulomb energy for these two charge distributions. A Coulomb energy that will depend on the angles theta and phi, that will depend on the orientation of the dumbbell, but as the problem has axial symmetry around the vertical axis, the only dependence that will really matter is the dependence on theta, there is no phi dependence here. So just apply Coulomb's law to express the interaction between one of the nuclear charges with the two electron charges, and the other nuclear charge with the two electron charges, and after some algebra you come to the expression that is here at the bottom of this slide. The constant C, you see its value here at the right, that's a positive constant. This expression depends on theta, on the orientation of the dumbbell. If you plot that expression, this energy as a function of theta, you find the orange full curve here. So the energy is lowest when the dumbbell has an angle of 0 or 180 degrees with the vertical axis, so when the dumbbell is vertical, the energy is maximal, when the dumbbell has an orientation that is perpendicular to the vertical axis. The dashed line, which you see here, would be the monopole contribution, if you would do a multipole expansion, but the full line that we have here is the exact solution, so no multipole expansion needed. And you see that the effect of the shape of the nucleus, the nucleus is more than a point, in this case it's a dumbbell, and we have taken into account the full complexity of the dumbbell in our exact solution. So the effect of this exact solution is that it lowers the energy for some orientations of the dumbbell, and raises the energy for other orientations of the dumbbell. This specific case, this toy model for our nucleus interacting with electrons, we will call toy model 0. Why 0? Because soon, and in a few weeks from now, there will be a toy model A and B as well. Let's immediately introduce toy model A, that's exactly the same as our toy model 0, except for the fact that we now put a small additional negative charge at the center of the dumbbell. So that is part of the electron system, it's a negative charge distribution, the total negative charge distribution now consists of 3 negative point charges, one of them being in the center of the nucleus, and you will probably see it already, this is a toy model that mimics the situation where some of the electron charge enters the nucleus, this minus epsilon negative charge is electron charge that is inside the nuclear volume. It's very easy to find what is the Coulomb energy contribution due to that negative charge that interacts with the positive charges of the nucleus, because the only quantity that matters there is the distance between that extra negative charge and the nuclear charges, and this distance is constant, whatever is the orientation of the nucleus, this distance is always the value L, which is one half of the diameter of the nucleus. So you can readily find that the extra energy, the extra Coulomb energy in toy model A that was not in toy model 0, is given by this additional term here at the right, a term that is constant, that does not depend on the angle theta. So in our graph here at the right, we can look at the dark blue curve, which is the energy for toy model A, and you will see that this is exactly the same shape as for the energy for toy model 0, but dropped over a constant value. In this way, we can even use that toy model A as a model for the isotope shift, because what is the effect of an isotope shift, of what is the physics behind an isotope shift? It means that the energy levels of the atom depend on the radius of the nucleus, let's take two of these toy atoms, with the same electron cloud, so the same dumble of negative charges, the same electron charge inside the nucleus, so the same minus epsilon, but two different nuclei, in the sense that they have the same charge, but a different radius. So the nucleus at the left has a diameter 2L1, the nucleus at the right has a diameter 2L2. So in this term here at the right, you will see a different contribution for the left nucleus, for isotope 1, than you see it for isotope 2. So these two classical toy atoms will have energy levels that are shifted by a constant, due to a different radius of their nuclei, a classical illustration of the isotope shift.