nuclear electric multipole moments

(automated transcription)

This is the list of properties, let's try to, well, to formalize how we get to some of these properties. And we will start with the properties that depend on the charge distribution inside the nucleus. So for the time being, my cartoon for a nucleus will be a general charge distribution. And deliberately the shape which you see here is very irregular. It can be any shape. I didn't tell in advance that this must be a positive charge distribution, because inside the quarks there are positive and negative charges, so this can be a mixture of positive and negative charges. The overall sum will be something positive, because the nucleus of course has a positive charge, but if you represent this by a continuous charge distribution, it's not necessarily positive at all points. And for sure in one of your courses in first or second bachelor year, you have seen a concept as a multipole expansion, where you could represent such an irregular object by a series of better behaved objects. Each of them has simpler properties, and if you sum the infinite series all together, you find the corresponding property of the complicated objects. What would that be for such a charge distribution? Well, the first few terms in your expansion would be the monopole term, where you represent this charge distribution by a point charge, with the same total charge as your original charge distribution had. That's the electric monopole. The electric dipole, which has no charge, but has a positive and a negative region separated from each other. And then if you combine two dipoles, you can get something that is called an electric quadrupole. And each of these, if you think in terms of electric potential for instance, each of these will contribute something to the electric potential of that entire object, but the deeper down you go in the series, the smaller the contribution will become. There will be on Minerva right away a few pages where you have the mathematical derivation of this. I'm sure that you have seen this before, but just to remind in a few pages how this technically worked, and you will find their derivation in Cartesian form, and shortly one in spherical form. We will come back in a few weeks on this spherical formula, that is the so-called Laplace expansion, which has some very useful properties for our purpose here. So this is the general equation that we will meet in a few weeks, in this PDF pages they don't use the general one, they use the far field situation, where you are rather far away from that irregular charge distribution. We will be interested in a few properties when you are really close to the nucleus, so for that reason we will need to use at some point that more general equation. So if you go through this mathematics, you will find expressions for this monopole contribution and the quadrupole contribution. Even in the classical derivation there will be no dipole contribution for nuclei, and we will see soon why. This monopole contribution, that's just the total charge of the nucleus, so not much problems in representing this, it's a scalar quantity, it's a single number. The quadrupole moment is mathematically more involved, that's not a scalar, it's a tensor, a tensor of rank 2, a spherical tensor of rank 2. So an object with 5 components, and in the literature you will find quite some ways to represent such a kind of object. You can represent it as a 3 by 3 traceless and symmetric matrix, so 9 different components, but because it is symmetric you have only 6 of them left, and because it should be traceless there are only 5 degrees of freedom left. That's a valid way to represent a quadrupole moment. Or you can represent it directly as a spherical tensor, so an object that behaves as the spherical harmonics, with rank l, which will be 2, and components m, which can be minus 2, minus 1, 0, plus 1, plus 2, so 5 components, here you have your 5 degrees of freedom for the quadrupole moment. Or you can combine these two representations and show these spherical components in a matrix shape that has already built in these properties of being traceless and symmetric. So each of these three representations you can meet in the literature about this. If you feel not completely familiar anymore about tensors, then there is an additional one inside that same PDF document, there is a section that explains the tensor concept from scratch, so if you need that you can catch up in that way. And you can test whether you understand it by making this little exercise, having in that PDF document the general equations for the monopole and the quadrupole moment. You can work out what is the monopole moment and the quadrupole moment for this planar charge distribution, for this three-dimensional charge distribution, both of them having only point charges, and for this continuous charge distribution, which is an actually symmetric ellipsoid that behaves according to this equation. So the message is so far, after applying this multipole expansion on the nuclear charge distribution, we have this monopole term and quadrupole term, this scalar property, the charge, and this rank two tensor property, the deviation from spherical symmetry. Now in practice, due to the fact that this nucleus has a spin, it will be really spinning about its symmetry axis. And that was a small question that was on one of the preceding slides, the one with the examples for iron-57, with the actual value of the spin of iron-57, you could in a classical model calculate what is the precession frequency of that nucleus. Just a very classical approximation. Do that once, and you will see how really fast nuclei are spinning. If you see how fast they spin, you will believe that every structure of the nucleus that is breaking the actual symmetry will be effectively washed out. If you would have a nucleus that is a cube, and it is spinning about an axis perpendicular to the cube, well, if it spins fast enough, what you observe is the behavior of a cylinder. That's why we don't need the full quadrupole moment tensor. These three objects, Qxx, Qyy, Qzz, which should sum to zero, that would be the quadrupole moments of a general object. If you look at it in an axis system that is designed to have maximal symmetry, we will call that the principal axis system, we will come to that in a few weeks. You have seen that in classical mechanics, for sure, as well. But for such a fast-spinning nucleus, because everything in the xy-plane is averaged out, the only thing you will need is that Qzz term, and that's just one number, and that is called the spectroscopic quadrupole moment. So very often in tabulations of nuclear properties, you will see for the quadrupole moment just one number, although it's a tensor, you will see one number, and that number is this one. It's a deviation from spherical symmetry along the axis of the spin, because any other deviation from spherical symmetry in the horizontal plane is wiped away due to this fast precession. So the quantities we will use to work with will be the charge of the nucleus, that is the point representation of the nucleus, and that is the quantity you have used so far, the deviation from spherical symmetry along the axis of the spin, and I deliberately write this here as an arrow and not as an ellipsoid, because in these two objects, in the point and in the deviation from spherical symmetry, there is no size information. A multipole expansion is about shape. It doesn't tell you so much about the absolute size. So that's an independent quantity, that's the, you could call this the mean square radius or the monopole radius, two very much related objects. Let's take that as the radius of the best approximating sphere for the nucleus. That contains the size information. So size, charge, and deviation from spherical symmetry. Yeah? When we talk about the size, we use the circle model, not the ellipsoid model. Well together, so the size of that circle, the degree of deformation, and the charge, that has to be distributed over the nuclear volume together, that gives you the properties of this ellipsoid. That's an, if the, so I have written here the general expression for an object that is actually symmetric, and it depends on this deformation parameters that you mentioned in the beginning, this beta 2, beta 4. If the deformation would be zero, if beta 2 and beta 4 are zero, you have a sphere with this radius. If you have just a quadrupole deformation, you have this beta 2 that behaves as a spherical harmonics of rank 2, but you can have more complex deformations that are still actually symmetric but not captured by the quadrupole term. That would be a beta 4, that's then the next step. After the quadrupole term, you have the octupole term, which is zero again, just as the dipole term, but there is the hexadecapole term coming next, which has the more complex deformation. But that's, that exists in nuclei, hexadecapole deformation, but it's very very small, and again for our purposes, quadrupole deformation is more than enough. If you would look through the nuclei where a quadrupole moment has been measured, you would see that this quantity, so, is typically growing with the size of the nucleus. And that you can find out yourself, when I asked to calculate a quadrupole moment for these three different objects, one of them was an ellipse with only a beta 2 term. That quadrupole moment would turn out to depend on beta 2 and on the radius of the nucleus, so the larger the radius, the heavier the nucleus, the more chance to have a large quadrupole moment. And so the same deformation beta in a small nucleus would produce a smaller quadrupole moment than in a larger nucleus. And that's what you see if you plot the quadrupole moments as a function of mass, they tend to be larger when the nucleus becomes heavier. And you even see some periodicity in this, which is due to the shell structure of nuclei. You have a kind of periodic table like effect in building up nuclei, just as you have in building up atoms. After a while you reach spherical symmetry and the story starts all over again. That's quadrupole moments, nuclear radii, if we need them we will take them from tabulations and there is one very nice paper, about 10 years old now, I will put that one on Minerva as well, where you have for every nucleus and every isotope that was measured at that time, where you have very precisely the nuclear radius. So this information we can consider as being known. That is the electric part of the story, so a nuclear charge distribution. Nuclei are more than only charge distributions, you have these protons are moving inside the nucleus and a moving charge, that's a current. Also the protons themselves, they have spin and spin in quantum mechanics is also related to a current. So in order to have a classical description of a nucleus, you do not need only the charge distribution, but also the current distribution. And you could repeat the entire multipole expansion for that current distribution. Which is analogous, but mathematically somewhat more involved, so we will certainly not go through that. I will just point to what is the mathematical complication here. In the case of a charge distribution, a static charge distribution, we know there is a scalar potential around that charge distribution. And the multipole expansion you make is a multipole expansion for that scalar potential. For a current distribution, you have a vector potential. And that is what complicates the multipole expansion. You can perfectly make that multipole expansion, but instead of products between scalar quantities in that multipole expansion, you will have dot products between vectors. And well, much heavier to write down, but the idea is the same. The symmetry properties are different. For the current multipole expansion, the odd terms will be the ones that survive. So the magnetic monopole moment of a nucleus will be zero, not surprising. The magnetic dipole moment will be non-zero. The magnetic quadrupole moment will be zero again. And the magnetic octopole moment will be very, very small. So the current property of a nucleus is mainly the magnetic dipole moment, a bar magnet. You represent a nucleus by a bar magnet. And the magnetic field of that bar magnet, that is the magnetic field due to that nucleus. So in the same cartoon, you would have here now a current distribution. And that can be approximated by a vector quantity, which is the magnetic dipole moment, which is the moment associated to a ring current. And then the next non-zero term, a magnetic octopole, four ring currents that are in a suitable configuration. And that would then be already a tensor of rank three.