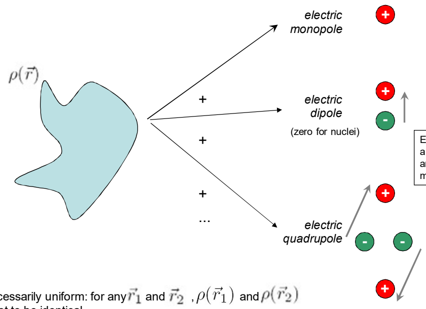
nuclear electric multipole moments

www. hyperfinecourse .org

electric nuclear multipole moments

A charge distribution of a general shape...

... can be written as a sum of multipole contributions :



Excercise: convince yourself that a dipole has no monopole moment, and that a quadrupole has no monopole and dipole moments.

Not necessarily uniform: for any $\vec{r}_1$ and $\vec{r}_2$, $\rho(\vec{r}_1)$ and $\rho(\vec{r}_2)$ need not to be identical.

2

electric nuclear multipole moments

Derivation in cartesian form : see pdf-pages

Derivation in spherical form :
- general formula:
- far-field case: see pdf-pages

$$\frac{1}{|\mathbf{r}_2 - \mathbf{r}_1|} = 4\pi \sum_{n,q} \frac{r_<^n}{r_>^{n+1}} \frac{1}{2n+1} Y_q^{n*}(\theta_1, \phi_1) Y_q^n(\theta_2, \phi_2)$$

Interpretation of monopole and quadrupole moments.

Different ways of representing the 5 degrees of freedom of the quadrupole moment tensor:
- As traceless and symmetric 3x3 matrix

$$\begin{bmatrix} Q_{xx} & Q_{xy} & Q_{xz} \\ Q_{xy} & Q_{yy} & Q_{yz} \\ Q_{xz} & Q_{yz} & Q_{zz} \end{bmatrix} \qquad Q_{xx} + Q_{yy} + Q_{zz} = 0$$

- As 5 components of a spherical tensor of rank 2

$$Q_m^\ell = eZ\sqrt{\frac{4\pi}{2\ell+1}} \langle I | r^\ell Y_m^\ell | I \rangle \qquad (\ell = 2)$$

- As a combination of both

$$Q_{ij} = \begin{bmatrix} Q_2^2 - \frac{1}{\sqrt{3}}Q_0^2 & Q_{-2}^2 & Q_1^2 \\ Q_{-2}^2 & -Q_2^2 - \frac{1}{\sqrt{3}}Q_0^2 & Q_{-1}^2 \\ Q_1^2 & Q_{-1}^2 & \frac{2}{\sqrt{3}}Q_0^2 \end{bmatrix}$$

Recommended reading: appendix on tensors.

3

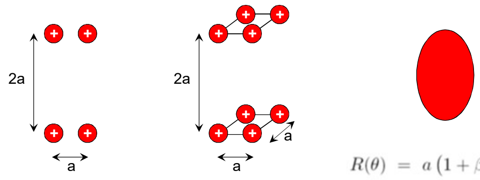
stefaan.cottenier@ugent.be

electric nuclear multipole moments

Exercise :

Apply the general equations to calculate the monopole, dipole and quadrupole moments of

this point charge configuration (2D) : *and this one (3D) :* *and this one (3D) :*



$R(\theta) = a\left(1 + \beta_2 Y_0^2(\theta)\right)$

(e.g. $\beta_2 = 0.2$)

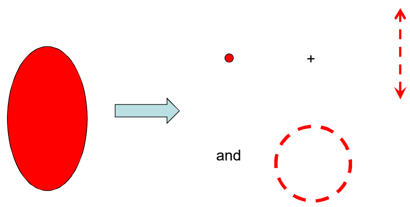
4

approximate shape of a nucleus: average radius and quadrupole deformation

The nucleus spins fast about its I-axis ➔ in axis system with z // I we have axial symmetry

$$\begin{bmatrix} -\frac{Q}{2} & 0 & 0 \\ 0 & -\frac{Q}{2} & 0 \\ 0 & 0 & Q \end{bmatrix}$$

The spectroscopic quadrupole moment Q (a scalar) says it all.



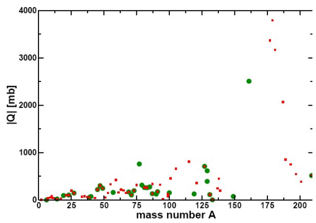
$R(\theta) = a\left(1 + \beta_2 Y_0^2(\theta) + \beta_4 Y_0^4(\theta) + \ldots\right)$

5

Trends in the (spectroscopic) quadrupole moment

$$eQ \simeq 3\sqrt{\frac{4\pi}{5}}\frac{eZ}{2\pi}\beta a^2$$

Q is large if:
- β is large (strongly deformed nuclei)
- a is large (heavy nuclei)



- oscillating behaviour due to nuclear shell structure
- increasing trend due to a-dependence

6

Trends in the mean square nuclear radius

Extensive experimental tabulation by I. Angeli (2004)

global summary of trend: $\sqrt{\langle r^2 \rangle} \; = \; 1.153 \, A^{0.294} \;\; \mathrm{fm}$

(just a snippet)

I. Angeli / Atomic Data and Nuclear Data Tables 87 (2004) 185–206 197

Table 1 (*continued*)

Z	El	A	R (fm)	$\Delta_{tot}R$ (fm)	$\Delta_{rel}R$ (fm)
		112	4.5950	.0020	
		113	4.6006	.0020	.00002
		114	4.6137	.0019	
		115	4.6170	.0054	.0049
		116	4.6284	.0021	.00002
		118	4.6316	.0037	.0024
		120	4.6379	.0059	.0045
49	In	104	4.5168	.0119	.0016
		105	4.5298	.0105	.0015
		106	4.5364	.0096	.0013
		107	4.5487	.0082	.0011
		108	4.5566	.0071	.0005
		109	4.5684	.0061	.0008
		110	4.5742	.0056	.0009
		111	4.5859	.0043	.0005
		112	4.5911	.0039	.0007
		113	4.6018	.0025	.00003
		114	4.6066	.0027	.0002
		115	4.6169	.0024	

Table 1 (*continued*)

Z	El	A	R (fm)	$\Delta_{tot}R$ (fm)	$\Delta_{rel}R$ (fm)
		126	4.7703	.0048	.0007
		128	4.7755	.0048	.0004
		129	4.7762	.0047	.0001
		130	4.7832	.0046	.0003
		131	4.7812	.0046	.0001
		132	4.7866	.0047	.0002
		134	4.7921	.0047	.0001
		136	4.7991	.0047	
		137	4.8143	.0048	.0003
		138	4.8359	.0054	.0003
		139	4.8511	.0060	.0006
		140	4.8694	.0067	.0002
		141	4.8845	.0075	.0004
		142	4.9016	.0086	.0009
		143	4.9137	.0092	.0004
		144	4.9300	.0102	.0005
		146	4.9575	.0119	.0005
55	Cs	118	4.7834	.0092	.0002
		119	4.7898	.0090	.0006

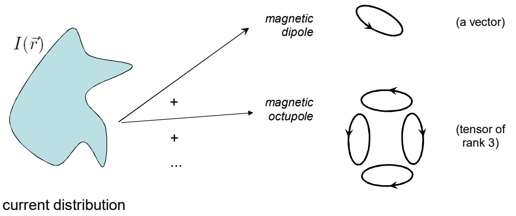
7

magnetic nuclear multipole moments

- What we discussed so far is the scalar potential due to a static charge distribution, developed into multipole components.
- A similar story applies to the vector potential due to a static current distribution, which can be developed into multipole components as well (mathematically a bit more involved).
- The parity of these magnetic nuclear multipole moments is different: odd terms survive.
- The first non-zero term is the magnetic dipole moment (a vector).
- The second non-zero is the magnetic octupole moment (a tensor of rank 3).

8

magnetic nuclear multipole moments



current distribution

9