why are odd electric moments zero ?

(automated transcription)

So I mentioned in passing that nuclei do not have electric dipole moments. We will spend a few minutes to that concept now and then from then on we will forget about these electric dipole moments and only the monopole and the quadrupole moment will matter. In order to understand that we have to look at the parity operator. A parity operator in three dimensions, what does it do? It replaces the X, everything that depends on X by minus X, Y by minus Y, Z by minus Z. It's an operator that has two real eigenvalues, plus one and minus one, and if you visually try to see what it does, and that's easiest to do in one dimension, so I take here in black a function f of X, I apply the parity operator on it, so X becomes minus X, and if you look at the graph what is f of minus X, that's the opposite of f of X. So after applying the parity operator, the black function becomes the red one, and it's somehow inverted. So you see, and you see it here, P of f, P working on f of X is minus f of X, so this is, that particular function is an eigenfunction of the parity operator with eigenvalue minus one. If I take another one, this black one here, and if I apply the parity operator, nothing will change. So that second function is also an eigenfunction of the parity operator, but with eigenvalue plus one. Why do we need to understand this? Well, we will make use of one particular property of eigenfunctions of the parity operator. If you integrate them over all space, then for those eigenfunctions that have eigenvalue one, minus one, that integral will be zero. And you see that right away, if you would integrate this black function over the linear axis, then the net surface area which you have is zero, as many positive contributions as negative contributions. That's a general property of odd parity eigenfunctions of the parity operator. Now in nuclear physics, the wave functions that describe the nucleus turn out to have a well-defined parity, which means they are eigenfunctions of the parity operator. There is no compelling reason why this must be the case, because you can easily draw many different functions that are not eigenfunctions of the parity operator. It's a very special property of being an eigenfunction of the parity operator. But somehow, observationally determined, wave functions of a nucleus are eigenfunctions of the parity operator. So we will use that experimental fact. Having this in mind, let's now write down the dipole moment for such a nucleus. And I just write down the x component of that dipole moment. The dipole moment is a vector, so three independent degrees of freedom. So I write here one of them. And in the general derivation, you would see that the x component of the dipole moment is the integral over space of x times the charge density distribution. Let's write that in a quantum mechanical picture, this density of the nuclear charge is the complex conjugates of the nuclear wave function times the nuclear wave function. And once you have it in that way, you can even immediately write it in a bracket notation. It's the matrix element of the x position operator squeezed between the nuclear state. Let's look back at these two integrals here. Nuclear wave functions have a well-defined parity, either even or odd. And parity will not change if you take the complex conjugate. So whatever is the parity, you have here psi star times psi, that has even parity, either parity plus one times plus one, or minus one times minus one. In both cases, the product, the density, has parity one. And this x operator here has odd parity, obviously. So this integrant has odd parity. So the integral over all space of something that has odd parity is zero. So due to the fact of having these well-defined parity states, a nucleus cannot have an electric dipole moment. It's a direct consequence of the well-defined parity properties of a nucleus. There is a second, yeah, question. It's not possible to find a wave function of a nucleus that isn't like a function of the parity operator? Not up to, well, I'm not a nuclear physicist, so I should be careful on what I'm saying. What I'm saying here, there is a hunt for parity violation in nuclei. But if that even ever has been found, it's a small effect. Certainly for the methods that we are using, parity violation does not play any role. There is a second explanation about this. I will not go through this argument now, but with the information that is here, you could reconstruct it. If you have troubles in reconstructing this, you can ask it next time, and then we will have more time to go through the details if there is a need for it.