the double ring

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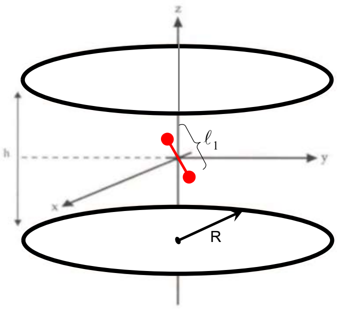
Example: a dumb-bell and a double ring.

total mass of dumbbell: m_1
total mass of double ring: m_2



$$\frac{r_1}{r_2} \ll 1 \quad \Rightarrow \quad \frac{l_1}{2} \ll \sqrt{\frac{h^2}{4} + R^2}$$

2

2

monopole energy :

$$E^{(0)}_{pot} = V^{(0)}_{sh} \cdot Q^{(0)}_{sh} = -\frac{G m_1 m_2}{\sqrt{\frac{h^2}{4} + R^2}}$$

quadrupole moment tensor of dumbbell:

$$_cQ^{(2)}_{sh} = \frac{3 m_1 l_1^2}{4} \begin{bmatrix} \sin^2\theta\cos^2\phi - \frac{1}{3} & \sin^2\theta\sin\phi\cos\phi & \sin\theta\cos\theta\cos\phi \\ \sin^2\theta\sin\phi\cos\phi & \sin^2\theta\sin^2\phi - \frac{1}{3} & \sin\theta\cos\theta\sin\phi \\ \sin\theta\cos\theta\cos\phi & \sin\theta\cos\theta\sin\phi & \cos^2\theta - \frac{1}{3} \end{bmatrix}$$

quadrupole field due to double ring:

$$_cV^{(2)}_{sh} = -\frac{G m_2 (h^2 - 2R^2)}{8R(R^2 + \frac{h^2}{4})^{\frac{5}{2}}} \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

(diagonal! ➔ PAS)

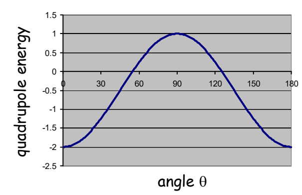
3

3

quadrupole energy :

(dot product between tensors = term by term multiplication, no matrix multiplication)

$$\frac{1}{6} Q^{(2)}_{sh} \cdot V^{(2)}_{sh} = -\underbrace{\frac{3 G m_1 m_2 l_1^2 (h^2 - 2R^2)}{32R(R^2 + \frac{h^2}{4})^{\frac{5}{2}}}}_{\alpha} (2\cos^2\theta - \sin^2\theta)$$



(picture made for α=1)

4

4

quadrupole energy :

(dot product between tensors = term by term multiplication, no matrix multiplication)

$$\frac{1}{6} Q^{(2)}_{sh} \cdot V^{(2)}_{sh} = -\underbrace{\frac{3 G m_1 m_2 l_1^2 (h^2 - 2R^2)}{32R(R^2 + \frac{h^2}{4})^{\frac{5}{2}}}}_{\alpha} (2\cos^2\theta - \sin^2\theta)$$

α > 0 α = 0 α < 0

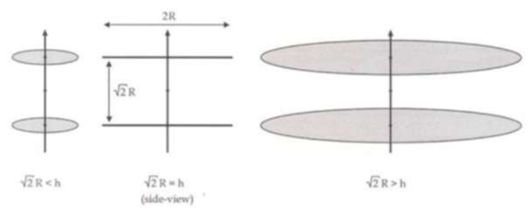


Fig. 2.6. *The three distinct classes of double-ring systems.*

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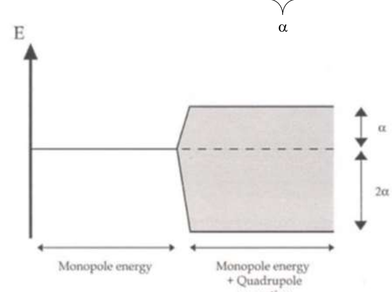
quadrupole energy :

(dot product between tensors = term by term multiplication, no matrix multiplication)

$$\frac{1}{6} Q^{(2)}_{sh} \cdot V^{(2)}_{sh} = -\underbrace{\frac{3 G m_1 m_2 l_1^2 (h^2 - 2R^2)}{32R(R^2 + \frac{h^2}{4})^{\frac{5}{2}}}}_{\alpha} (2\cos^2\theta - \sin^2\theta)$$



(picture for α > 0)

6

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